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Transient motion of an interfacial line force or dislocation in an anisotropic elastic bimaterial

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Abstract

A full-field dynamic solution of an interfacial line force or dislocation in bonded anisotropic elastic half-spaces is presented. The form of the solution resembles that of the corresponding static solution. The evaluation of the solution requires only the calculation of certain eigenvalue problems. Particular attention is given to the singular feature in the response associated with the interfacial Stoneley wave. Numerical examples are given to illustrate the characteristics of a pair of bonded misoriented half-spaces of GaAs.

Keywords: Interfacial dislocation; Interfacial force; Transient problem; Wave propagation; Anisotropic material

1. Introduction

A prominent feature of the dynamics of the interface separating two dissimilar elastic media is the possible existence of interfacial waves, commonly referred to as Stoneley waves. Such waves propagate without dispersion along the interface and attenuate exponentially with distance normal to the interface in both media. The early investigation of interfacial waves was primarily for geophysical applications. More recently interfacial waves have been applied to non-destructive materials characterization.

Stoneley (1924) was the first to demonstrate that steady interfacial waves do not always exist for bonded isotropic half-spaces. Scholte (1947) showed that for bonded isotropic half-spaces the interfacial waves only exist under very severe restrictions on material constants. Barnett and Lothe (1974) and Chadwick and Currie (1974) deduced a general condition determining the Stoneley wave velocity in bonded anisotropic half-spaces. Barnett et al. (1985) further established the uniqueness and existence conditions of subsonic Stoneley waves in bonded anisotropic half-spaces. Every and Briggs (1998) presented algorithms based on integral transforms for calculating the time domain displacement response of bonded anisotropic half-spaces to impulsive line and point forces at their interface. The calculation of the line force response at the interface was reduced to a purely algebraic problem.

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In this paper the transient motion of an anisotropic bimaterial due to a line force and a line dislocation at the interface is considered. A formulation for two-dimensional self-similar problems in elastodynamics developed by Wu (2000) is employed. In this formulation the solution is expressed in terms of the eigenvalues and eigenvectors of a six-dimensional matrix, which is a function of the material constants, time and position. A major advantage of the proposed formulation is that no integral transforms are required. This fact greatly facilitates derivations of explicit solutions. Indeed, a *full-field* solution of the problem under consideration is derived. Particular attention is given to the singular feature in the response associated with the interfacial Stoneley wave. Numerical examples are presented to illustrate the characteristics of a pair of bonded misoriented half-spaces of GaAs.

2. Formulation

For two-dimensional deformation in which the Cartesian components of the stress σ_{ij} and the displacement u_i, $i,j = 1,2,3$, are independent of x_3, the equations of motion are

$$(\mathbf{t}_{1,1} + \mathbf{t}_{2,2}) = \rho\ddot{\mathbf{u}}, \tag{1}$$

where $\mathbf{t}_1 = (\sigma_{11}, \sigma_{21}, \sigma_{31})^{\mathrm{T}}$, $\mathbf{t}_2 = (\sigma_{12}, \sigma_{22}, \sigma_{32})^{\mathrm{T}}$, $\ddot{\mathbf{u}}$ is the acceleration, ρ is the density, a subscript comma denotes partial differentiation with respect to coordinates and an overhead dot designates derivative with respect to time t. The stress–strain laws are

$$\mathbf{t}_1 = \mathbf{Q}\mathbf{u}_{,1} + \mathbf{R}\mathbf{u}_{,2}, \tag{2}$$

$$\mathbf{t}_2 = \mathbf{R}^{\mathrm{T}}\mathbf{u}_{,1} + \mathbf{T}\mathbf{u}_{,2}, \tag{3}$$

where the matrices $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{T}$ are related to the elastic constants C_{ijks} by

$$Q_{ik} = C_{i1k1}, \quad R_{ik} = C_{i1k2}, \quad T_{ik} = C_{i2k2}.$$

The equations of motion expressed in terms of the displacements are obtained by substituting Eqs. (2) and (3) into Eq. (1) as

$$\mathbf{Q}\mathbf{u}_{,11} + (\mathbf{R} + \mathbf{R}^{\mathrm{T}})\mathbf{u}_{,12} + \mathbf{T}\mathbf{u}_{,22} = \rho\ddot{\mathbf{u}}. \tag{4}$$

Let the displacement be assumed as $\mathbf{u}(x_1, x_2, t) = \mathbf{u}(\omega)$ with the variable $\omega(x_1, x_2, t)$ implicitly defined by

$$\omega t = x_1 + p(\omega)x_2. \tag{5}$$

Eq. (4) becomes (Wu, 2000)

$$\frac{\partial}{\partial\omega}\left\{\frac{\partial\omega}{\partial x_1}[\mathbf{Q} - \rho\omega^2\mathbf{I} + p(\omega)(\mathbf{R} + \mathbf{R}^{\mathrm{T}}) + p(\omega)^2\mathbf{T}]\mathbf{u}'(\omega)\right\} = \mathbf{0}, \tag{6}$$

where $\mathbf{I}$ is the identity matrix and $\partial\omega/\partial x_1 = 1/(t - p'(\omega)x_2)$. Let $\mathbf{u}'(\omega)$ be expressed as

$$\mathbf{u}'(\omega) = f(\omega)\mathbf{a}(\omega), \tag{7}$$

where $f(\omega)$ is an arbitrary scalar function of ω. It follows that $\mathbf{u}(\omega)$ is a solution of Eq. (4) if

$$\mathbf{D}(p, \omega)\mathbf{a}(\omega) = \mathbf{0}, \tag{8}$$

where $\mathbf{D}(p, \omega)$ is given by

$$\mathbf{D}(p, \omega) = \mathbf{Q} + p(\mathbf{R} + \mathbf{R}^{\mathrm{T}}) + p^2\mathbf{T} - \rho\omega^2\mathbf{I}. \tag{9}$$

For non-trivial solutions of $\mathbf{a}(\omega)$ we must have

$$|\mathbf{D}(p,\omega)| = 0, \tag{10}$$

where $|\mathbf{D}|$ is the determinant of $\mathbf{D}$.

Eq. (10) provides six eigenvalues of p as a function of ω, denoted by $p_k(\omega)$, $k = 1,2,\ldots,6$. The function $p_k(\omega)$ is single-valued if ω is allowed to range over the six sheets Σ^k of its Riemann surface, taking the values $p_k(\omega)$ on Σ^k (Willis, 1973). If ω is real and $|\omega|$ is sufficiently large, there are six real roots $p_k(\omega)$. Three of these roots are characterized by $p'(\omega) > 0$ and the other three by $p'(\omega) < 0$. The three of the former type will be assigned to the Riemann surfaces Σ^k $(k = 1,2,3)$ and the three of the latter type to Σ^k $(k = 4,5,6)$. The sheets are connected across appropriate lines joining the branch points of $p_k(\omega)$, which are located on the real axis in the complex ω-plane and are determined by $\omega'(p) = 0$. For a real value of ω, $\omega = y_1 + p(\omega)y_2$ represents a plane wave front which is tangent to a wavefront surface at $(\omega - p(\omega)/p'(\omega), 1/p'(\omega))$ (Wu, 2000). Thus real p_k $(k = 1,2,3)$ are associated with the rays propagating in the direction of positive x_2 while p_k $(k = 4,5,6)$ with the rays propagating in the negative direction of x_2. It can be shown that complex $p_k(\omega)$ has positive imaginary part in the upper half of Σ^k $(k = 1,2,3)$ and negative imaginary part in the upper plane of Σ^k $(k = 4,5,6)$. The variable $\omega_k = \omega_k(x_1,x_2,t)$ can then be solved from Eq. (5) by taking $p(\omega) = p_k(\omega)$.

From Eq. (5) the complex variables ω_k may be written as

$$\omega_k = y_1 + p_k(\omega_k)y_2, \tag{11}$$

where $y_\alpha = x_\alpha/t$, $\alpha = 1,2$. Substitution of Eq. (11) into Eq. (9) leads to

$$\mathbf{D} = \widehat{\mathbf{Q}} + p(\widehat{\mathbf{R}} + \widehat{\mathbf{R}}^{\mathrm{T}}) + p^2\widehat{\mathbf{T}}, \tag{12}$$

where

$$\widehat{Q}_{ik} = \widehat{C}_{i1k1}, \quad \widehat{R}_{ik} = \widehat{C}_{i1k2}, \quad \widehat{T}_{ik} = \widehat{C}_{i2k2},$$

and $\widehat{C}_{ijks} = C_{ijks} - \rho y_j y_s \delta_{ik}$. Thus p_k as a function of y_1 and y_2 can be obtained from Eq. (10) with $\mathbf{D}$ given by Eq. (12). Once $p_k(y_1,y_2)$ are obtained, $\omega_k(y_1,y_2)$ are simply given by Eq. (11). Note that as $t \to \infty$, the eigenvalues p_k reduce to Stroh's eigenvalues for anisotropic elastostatics (Stroh, 1958).

From Eq. (7), the general solution of Eq. (4) may be represented as

$$\mathbf{u}(x_1,x_2,t)_{,1} = 2\,\mathrm{Re}\left\{\sum_k \frac{\partial\omega_k}{\partial x_1} f_k(\omega_k)\mathbf{a}_k(\omega_k)\right\}, \tag{13}$$

$$\mathbf{u}(x_1,x_2,t)_{,2} = 2\,\mathrm{Re}\left\{\sum_k \frac{\partial\omega_k}{\partial x_2} f_k(\omega_k)\mathbf{a}_k(\omega_k)\right\}, \tag{14}$$

$$\dot{\mathbf{u}}(x_1,x_2,t) = 2\,\mathrm{Re}\left\{\sum_k \frac{\partial\omega_k}{\partial t} f_k(\omega_k)\mathbf{a}_k(\omega_k)\right\}, \tag{15}$$

where $k = 1,2,3$ or $4,5,6$, and

$$\frac{\partial\omega_k}{\partial x_1} = \frac{1}{t - p_k'(\omega_k)},$$
$$\frac{\partial\omega_k}{\partial x_2} = p_k(\omega_k)\frac{\partial\omega_k}{\partial x_1},$$
$$\frac{\partial\omega_k}{\partial t} = -\omega_k\frac{\partial\omega_k}{\partial x_1}.$$

The choice of the range of k depends on whether up-going rays or down-going rays are considered.

By substituting Eqs. (13) and (14) into Eqs. (2) and (3), the general solutions of the stress vectors $\mathbf{t}_1$ and $\mathbf{t}_2$ can be expressed as

$$\mathbf{t}_1(x_1,x_2,t) = -2\,\mathrm{Re}\left\{\sum_k f_k(\omega_k)\left(\rho\omega_k\frac{\partial\omega_k}{\partial t}\mathbf{a}_k(\omega_k) + \frac{\partial\omega_k}{\partial x_2}\mathbf{b}_k(\omega_k)\right)\right\}, \tag{16}$$

$$\mathbf{t}_2(x_1,x_2,t) = 2\,\mathrm{Re}\left\{\sum_k \frac{\partial\omega_k}{\partial x_1} f_k(\omega_k)\mathbf{b}_k(\omega_k)\right\}, \tag{17}$$

where

$$\mathbf{b}_k(\omega) = (\mathbf{R}^{\mathrm{T}} + p_k(\omega)\mathbf{T})\mathbf{a}_k(\omega). \tag{18}$$

A useful expression for $p_k'(\omega_k)$ is given by (Wu, 2000)

$$p_k'(\omega) = \rho\omega\frac{\mathbf{a}_k(\omega)^{\mathrm{T}}\mathbf{a}_k(\omega)}{\mathbf{a}_k(\omega)^{\mathrm{T}}\mathbf{b}_k(\omega)}.$$

3. Full-field solution

Consider a bimaterial consisting of two dissimilar elastic half-spaces bonded together. Let the half-space $x_2 \geqslant 0$ be occupied by material 1 and the half-space $x_2 \leqslant 0$ be occupied by material 2. The bimaterial is initially stress-free and is subjected to a line force $H(t)\mathbf{F}$ and a dislocation of Burgers vector $H(t)\mathbf{b}$ at the origin for $t > 0$. Here H is the Heaviside step function. The associated jump conditions at the interface $x_2 = 0$ are given by

$$\mathbf{u}_{,1}(x_1,0^+,t) - \mathbf{u}^*_{,1}(x_1,0^-,t) = -\delta(x_1)H(t)\mathbf{b}, \tag{19}$$

$$\mathbf{t}_2(x_1,0^+,t) - \mathbf{t}^*_2(x_1,0^-,t) = -\delta(x_1)H(t)\mathbf{F}, \tag{20}$$

where the superscript "$*$" denotes quantities referred to material 2.

Since up-going waves are generated in material 1 and down-going waves are in material 2, the expressions for $\mathbf{u}_{,1}$ and $\mathbf{t}_2$ in materials 1 are given by

$$\mathbf{u}_{,1} = 2\,\mathrm{Re}\left[\mathbf{A}(\omega)\left\langle\frac{\partial\omega}{\partial x_1}\right\rangle\mathbf{f}(\omega)\right], \tag{21}$$

$$\mathbf{t}_2 = 2\,\mathrm{Re}\left[\mathbf{B}(\omega)\left\langle\frac{\partial\omega}{\partial x_1}\right\rangle\mathbf{f}(\omega)\right], \tag{22}$$

where

$$\begin{aligned}
\mathbf{A}(\omega) &= [\mathbf{a}_1(\omega_1), \mathbf{a}_2(\omega_2), \mathbf{a}_3(\omega_3)],\\
\mathbf{B}(\omega) &= [\mathbf{b}_1(\omega_1), \mathbf{b}_2(\omega_2), \mathbf{b}_3(\omega_3)],\\
\left\langle\frac{\partial\omega}{\partial x_1}\right\rangle &= \mathrm{diag}\left[\frac{\partial\omega_1}{\partial x_1}, \frac{\partial\omega_2}{\partial x_1}, \frac{\partial\omega_3}{\partial x_1}\right],\\
\mathbf{f}(\omega) &= [f_1(\omega_1), f_2(\omega_2), f_3(\omega_3)]^{\mathrm{T}},
\end{aligned}$$

and "diag" denotes diagonal matrix. Those for material 2 are

$$\mathbf{u}^*_{,1} = 2\,\mathrm{Re}\left[\mathbf{A}^*(\omega^*)\left\langle \frac{\partial\omega^*}{\partial x_1} \right\rangle \mathbf{f}^*(\omega^*)\right], \tag{23}$$

$$\mathbf{t}^*_2 = 2\,\mathrm{Re}\left[\mathbf{B}^*(\omega^*)\left\langle \frac{\partial\omega^*}{\partial x_1} \right\rangle \mathbf{f}^*(\omega^*)\right], \tag{24}$$

where

$$\mathbf{A}^*(\omega^*) = [\mathbf{a}^*_4(\omega^*_4), \mathbf{a}^*_5(\omega^*_5), \mathbf{a}^*_6(\omega^*_6)],$$
$$\mathbf{B}^*(\omega^*) = [\mathbf{b}^*_4(\omega^*_4), \mathbf{b}^*_5(\omega^*_5), \mathbf{b}^*_6(\omega^*_6)],$$
$$\mathbf{f}^*(\omega^*) = [f^*_4(\omega^*_4), f^*_5(\omega^*_5), f^*_6(\omega^*_6)]^{\mathrm{T}},$$
$$\left\langle \frac{\partial\omega^*}{\partial x_1} \right\rangle = \mathrm{diag}\left[\frac{\partial\omega^*_4}{\partial x_1}, \frac{\partial\omega^*_5}{\partial x_1}, \frac{\partial\omega^*_6}{\partial x_1}\right].$$

The forms of $\mathbf{f}(\omega)$ and $\mathbf{f}^*(\omega^*)$ are assumed as follows:

$$\mathbf{f}(\omega) = \frac{1}{2\pi\mathrm{i}}\left\langle \frac{1}{\omega} \right\rangle \mathbf{q}(\omega), \tag{25}$$

$$\mathbf{f}^*(\omega^*) = \frac{1}{2\pi\mathrm{i}}\left\langle \frac{1}{\omega^*} \right\rangle \mathbf{q}^*(\omega^*), \tag{26}$$

where

$$\left\langle \frac{1}{\omega} \right\rangle = \mathrm{diag}\left[\frac{1}{\omega_1}, \frac{1}{\omega_2}, \frac{1}{\omega_3}\right], \quad \mathbf{q}(\omega) = [q_1(\omega_1), q_2(\omega_2), q_3(\omega_3)]^{\mathrm{T}},$$

$$\left\langle \frac{1}{\omega^*} \right\rangle = \mathrm{diag}\left[\frac{1}{\omega^*_4}, \frac{1}{\omega^*_5}, \frac{1}{\omega^*_6}\right], \quad \mathbf{q}^*(\omega) = [q^*_4(\omega^*_4), q^*_5(\omega^*_5), q^*_6(\omega^*_6)]^{\mathrm{T}},$$

q_k and q^*_k are analytic at $\omega_k = 0$ and $\omega^*_{k+3} = 0$, $k = 1, 2, 3$, respectively. With Eq. (25) substituted, Eqs. (21) and (22) become

$$\mathbf{u}_{,1} = \frac{1}{\pi}\mathrm{Im}\left[\mathbf{A}(\omega)\left\langle \frac{1}{\omega}\frac{\partial\omega}{\partial x_1} \right\rangle \mathbf{q}(\omega)\right], \tag{27}$$

$$\mathbf{t}_2 = \frac{1}{\pi}\mathrm{Im}\left[\mathbf{B}(\omega)\left\langle \frac{1}{\omega}\frac{\partial\omega}{\partial x_1} \right\rangle \mathbf{q}(\omega)\right]. \tag{28}$$

With Eq. (26) substituted, Eqs. (23) and (24) become

$$\mathbf{u}^*_{,1} = \frac{1}{\pi}\mathrm{Im}\left[\mathbf{A}^*(\omega^*)\left\langle \frac{1}{\omega^*}\frac{\partial\omega^*}{\partial x_1} \right\rangle \mathbf{q}^*(\omega^*)\right], \tag{29}$$

$$\mathbf{t}^*_2 = \frac{1}{\pi}\mathrm{Im}\left[\mathbf{B}^*(\omega^*)\left\langle \frac{1}{\omega^*}\frac{\partial\omega^*}{\partial x_1} \right\rangle \mathbf{q}^*(\omega^*)\right]. \tag{30}$$

As $x_2 \to 0^+$, $\omega_k = \eta = y_1 + \mathrm{i}0^+$, $k = 1, 2, 3$, Eqs. (27) and (28) yield

$$\mathbf{u}_{,1} = \frac{1}{\pi x_1}\mathrm{Im}[\mathbf{A}(\eta)\mathbf{q}(\eta)] - \frac{\delta(y_1)}{t}\mathrm{Re}[\mathbf{A}(\eta)\mathbf{q}(\eta)], \tag{31}$$

$$\mathbf{t}_2 = \frac{1}{\pi x_1}\mathrm{Im}[\mathbf{B}(\eta)\mathbf{q}(\eta)] - \frac{\delta(y_1)}{t}\mathrm{Re}[\mathbf{B}(\eta)\mathbf{q}(\eta)]. \tag{32}$$

Similarly as $x_2 \to 0^-$, $\omega_k^* = \eta = y_1 + \mathrm{i}0^+$, $k = 4, 5, 6$, Eqs. (29) and (30) give

$$\mathbf{u}^*_{,1} = \frac{1}{\pi x_1}\mathrm{Im}[\mathbf{A}^*(\eta)\mathbf{q}^*(\eta)] - \frac{\delta(y_1)}{t}\mathrm{Re}[\mathbf{A}^*(\eta)\mathbf{q}^*(\eta)], \tag{33}$$

$$\mathbf{t}^*_2 = \frac{1}{\pi x_1}\mathrm{Im}[\mathbf{B}^*(\eta)\mathbf{q}^*(\eta)] - \frac{\delta(y_1)}{t}\mathrm{Re}[\mathbf{B}^*(\eta)\mathbf{q}^*(\eta)]. \tag{34}$$

In Eqs. (31)–(34) the following identity has been applied:

$$\frac{1}{\eta} = \frac{1}{y_1} - \mathrm{i}\pi\delta(y_1).$$

Substitution of Eqs. (31)–(34) into Eqs. (19) and (20) leads to

$$\mathrm{Im}[\mathbf{A}(\eta)\mathbf{q}(\eta)] = \mathrm{Im}[\mathbf{A}^*(\eta)\mathbf{q}^*(\eta)], \quad \mathrm{Im}[\mathbf{B}(\eta)\mathbf{q}(\eta)] = \mathrm{Im}[\mathbf{B}^*(\eta)\mathbf{q}^*(\eta)],$$
$$\mathrm{Re}[\mathbf{A}(\eta)\mathbf{q}(\eta)] - \mathrm{Re}[\mathbf{A}^*(\eta)\mathbf{q}^*(\eta)] = \mathbf{b}, \quad \mathrm{Re}[\mathbf{B}(\eta)\mathbf{q}(\eta)] - \mathrm{Re}[\mathbf{B}^*(\eta)\mathbf{q}^*(\eta)] = \mathbf{F},$$

or simply

$$\mathbf{A}(\eta)\mathbf{q}(\eta) - \mathbf{A}^*(\eta)\mathbf{q}^*(\eta) = \mathbf{b}, \tag{35}$$

$$\mathbf{B}(\eta)\mathbf{q}(\eta) - \mathbf{B}^*(\eta)\mathbf{q}^*(\eta) = \mathbf{F}. \tag{36}$$

The solutions of $\mathbf{q}(\eta)$ and $\mathbf{q}^*(\eta)$ of Eqs. (35) and (36) may be expressed as

$$\mathbf{q}(\eta) = -\mathbf{A}(\eta)^{-1}\mathbf{M}(\eta)^{-1}\left(\mathbf{M}_2^*(\eta)^{-1}\mathbf{b} + \mathrm{i}\mathbf{F}\right), \tag{37}$$

$$\mathbf{q}^*(\eta) = -\mathbf{A}^*(\eta)^{-1}\mathbf{M}(\eta)^{-1}\left(\mathbf{M}_1(\eta)^{-1}\mathbf{b} + \mathrm{i}\mathbf{F}\right), \tag{38}$$

where $\mathbf{M}_1 = -\mathrm{i}\mathbf{B}(\eta)\mathbf{A}(\eta)^{-1}$, $\mathbf{M}_2^* = -\mathrm{i}\mathbf{B}^*(\eta)\mathbf{A}^*(\eta)^{-1}$ are the impedance tensors (Lothe and Barnett, 1976) of material 1 and material 2, respectively, and $\mathbf{M}$ is given by

$$\mathbf{M}(\eta) = \mathbf{M}_1 - \mathbf{M}_2^*. \tag{39}$$

The functions $\mathbf{q}(\omega)$ and $\mathbf{q}^*(\omega^*)$ are obtained from $\mathbf{q}(\eta)$ and $\mathbf{q}^*(\eta)$ by

$$\mathbf{q}(\omega) = \sum_{k=1}^{3}\mathbf{I}_k\mathbf{q}(\omega_k), \quad \mathbf{q}(\omega) = \sum_{k=1}^{3}\mathbf{I}_k\mathbf{q}^*(\omega^*_{k+3}),$$

where $\mathbf{I}_1 = \mathrm{diag}[1, 0, 0]$, $\mathbf{I}_2 = \mathrm{diag}[0, 1, 0]$, and $\mathbf{I}_3 = \mathrm{diag}[0, 0, 1]$.

If material 1 and material 2 are identical, the problem reduces to the one for an infinite homogeneous solid. In this case the following identities exist (Ting, 1996, p. 445):

$$\mathbf{B}(\eta)\left\langle\frac{1}{\gamma}\right\rangle\mathbf{A}(\eta)^{\mathrm{T}} + \mathbf{B}^*(\eta)\left\langle\frac{1}{\gamma^*}\right\rangle\mathbf{A}^*(\eta)^{\mathrm{T}} = \mathbf{I}, \tag{40}$$

$$\mathbf{A}(\eta)\left\langle \frac{1}{\gamma} \right\rangle \mathbf{A}(\eta)^{\mathrm{T}} + \mathbf{A}^*(\eta)\left\langle \frac{1}{\gamma^*} \right\rangle \mathbf{A}^*(\eta)^{\mathrm{T}} = \mathbf{0}, \tag{41}$$

$$\mathbf{A}(\eta)^{\mathrm{T}}\mathbf{B}^*(\eta) + \mathbf{B}(\eta)^{\mathrm{T}}\mathbf{A}^*(\eta) = \mathbf{0}, \tag{42}$$

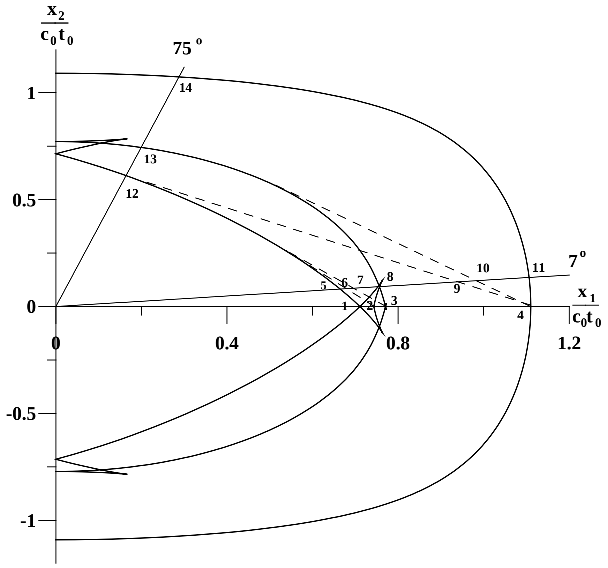

Fig. 1. Wave fronts and angles of observation for the 10°/−10° GaAs bimaterial.

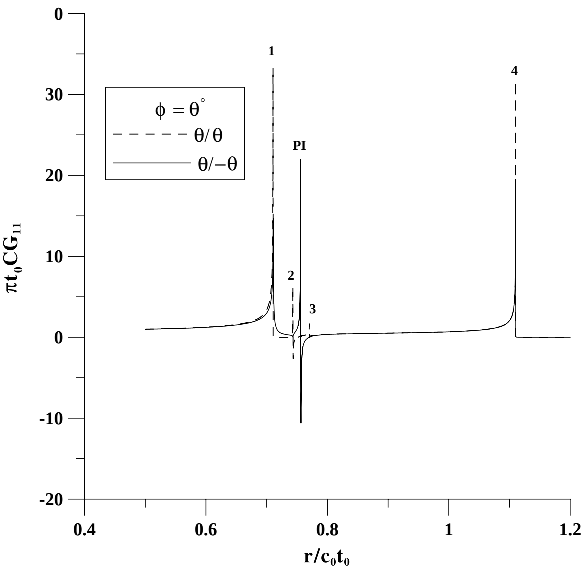

Fig. 2. G_{11} as a function of r for $\phi = 0°$.

where

$$\left\langle \frac{1}{\gamma} \right\rangle = \mathrm{diag}\left[\frac{1}{\gamma_1}, \frac{1}{\gamma_2}, \frac{1}{\gamma_2}\right], \quad \left\langle \frac{1}{\gamma^*} \right\rangle = \mathrm{diag}\left[\frac{1}{\gamma_4}, \frac{1}{\gamma_5}, \frac{1}{\gamma_6}\right], \quad \text{and} \quad \gamma_k = 2\mathbf{a}_k^{\mathrm{T}}\mathbf{b}_k, \;\; k = 1, 2, \ldots, 6.$$

Using Eqs. (40) and (41), the matrix $\mathbf{M}$ of Eq. (39) becomes

$$\mathbf{M} = -\mathrm{i}\left(\mathbf{A}(\eta)\left\langle \frac{1}{\gamma} \right\rangle \mathbf{A}(\eta)^{\mathrm{T}}\right)^{-1}. \tag{43}$$

Substitution Eq. (43) into Eq. (37) and using Eq. (42) leads to

$$\mathbf{q}(\eta) = \left\langle \frac{1}{\gamma} \right\rangle \left(\mathbf{A}(\eta)^{\mathrm{T}}\mathbf{F} + \mathbf{B}^{\mathrm{T}}(\eta)\mathbf{b}\right). \tag{44}$$

Eq. (44) recovers the solution given by Wu (2000). The solution for Lamb's problem may also be obtained by setting $\mathbf{M}_2^*(\eta) \to \mathbf{0}$. In doing so, $\mathbf{M}$ simplifies to

$$\mathbf{M}(\eta) = -\mathrm{i}\mathbf{B}(\eta)\mathbf{A}(\eta)^{-1}$$

and $\mathbf{q}(\eta)$ to

$$\mathbf{q}(\eta) = \mathbf{B}(\eta)^{-1}\mathbf{F}. \tag{45}$$

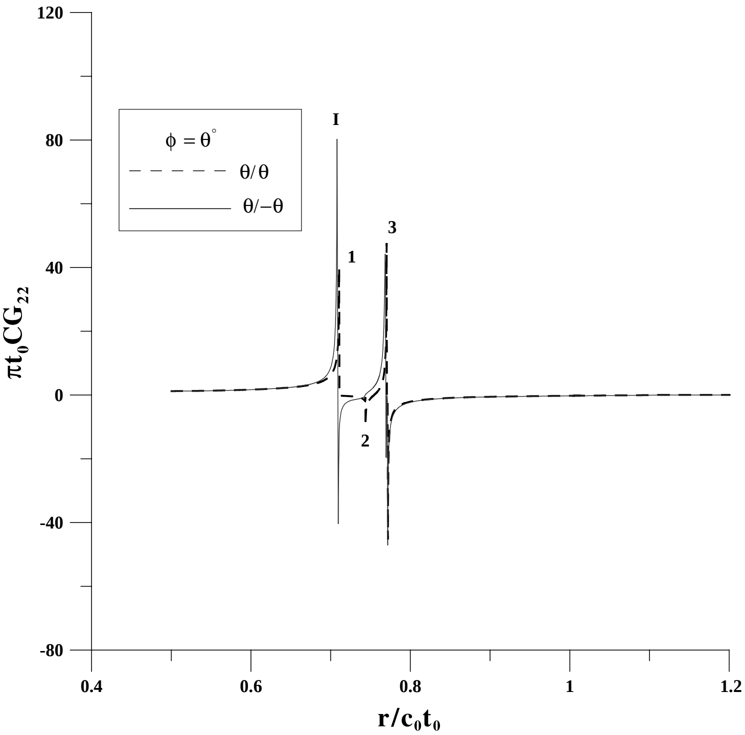

Fig. 3. G_{22} as a function of r for $\phi = 0°$.

Eq. (45) is the same as that derived by Wu (2000). Finally in the limit as $t \to \infty$, $p_k(0)$ appear as three pairs of complex conjugate constants and $\omega_k = z_k/t$, where $z_k = x_1 + p_k(0)x_2$. the result derived here reduce to that for the corresponding static problem (Ting, 1996, pp. 273–283).

The particle velocities $\dot{\mathbf{u}}$ and $\dot{\mathbf{u}}^*$ in materials 1 and 2, respectively, are given by substituting Eqs. (25) and (26) into Eq. (15) as

$$\dot{\mathbf{u}} = -\frac{1}{\pi}\mathrm{Im}\left[\mathbf{A}(\omega)\left\langle\frac{\partial\omega}{\partial x_1}\right\rangle\mathbf{q}(\omega)\right], \tag{46}$$

$$\dot{\mathbf{u}}^* = -\frac{1}{\pi}\mathrm{Im}\left[\mathbf{A}^*(\omega^*)\left\langle\frac{\partial\omega^*}{\partial x_1}\right\rangle\mathbf{q}^*(\omega^*)\right]. \tag{47}$$

If only the line force is considered, Eqs. (46) and (47) may be expressed as

$$\dot{\mathbf{u}} = \mathbf{GF}, \quad \dot{\mathbf{u}}^* = \mathbf{G}^*\mathbf{F},$$

where

$$\mathbf{G} = \frac{1}{\pi}\mathrm{Re}\left[\mathbf{A}(\omega)\left\langle\frac{\partial\omega}{\partial x_1}\right\rangle\sum_{k=1}^{3}\mathbf{I}_k\mathbf{A}(\omega_k)^{-1}\mathbf{M}(\omega_k)^{-1}\right], \tag{48}$$

$$\mathbf{G}^* = \frac{1}{\pi}\mathrm{Re}\left[\mathbf{A}^*(\omega^*)\left\langle\frac{\partial\omega^*}{\partial x_1}\right\rangle\sum_{k=1}^{3}\mathbf{I}_k\mathbf{A}^*(\omega^*_{k+3})^{-1}\mathbf{M}(\omega^*_{k+3})^{-1}\right]. \tag{49}$$

Note that $\dot{\mathbf{u}}$ and $\dot{\mathbf{u}}^*$ are also the displacements due to an *impulsive* interfacial line force and $\mathbf{G}$ and $\mathbf{G}^*$ are the corresponding Green's functions.

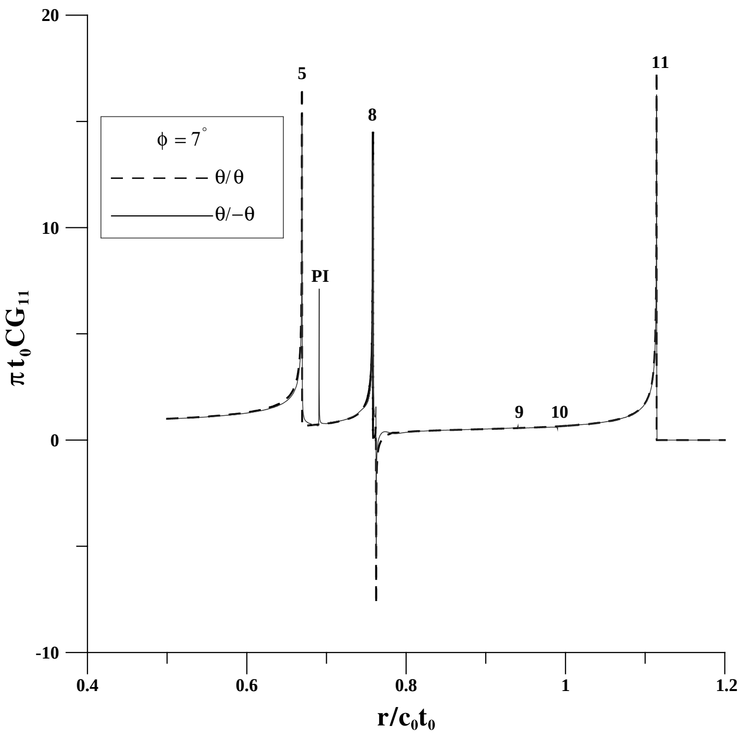

Fig. 4. G_{11} as a function of r for $\phi = 7°$.

4. Interfacial Stoneley waves

From Eq. (46) the velocity $\dot{\mathbf{u}}$ at the interface is given by

$$\dot{\mathbf{u}} = \frac{1}{\pi t}\mathrm{Im}[\mathbf{M}(\eta)^{-1}(\mathbf{M}_2^*(\eta)^{-1}\mathbf{b} + \mathrm{i}\mathbf{F})]. \tag{50}$$

The matrix $\mathbf{M}(\eta)^{-1}$ may be expressed as

$$\mathbf{M}(\eta)^{-1} = \frac{1}{m(\eta)}\mathrm{adj}(\mathbf{M}(\eta)), \tag{51}$$

where $m(\eta)$ is the determinant of $\mathbf{M}(\eta)$ and $\mathrm{adj}(\mathbf{M}(\eta))$ is the adjoint matrix of $\mathbf{M}(\eta)$. Chadwick and Currie (1974) showed that interfacial Stoneley waves exist if there are speeds v_s such that

$$m(v_s) = 0. \tag{52}$$

Let $\hat{v}_1$ and $\hat{v}_2$ be the smallest bulk wave speeds associated with materials 1 and 2, respectively. Barnett et al. (1985) showed that if v_s is subsonic, i.e., $0 < v_s < \min[\hat{v}_1, \hat{v}_2]$, it is unique. They also showed that v_s is not less than the smaller of the surface wave speeds of the two materials. A subsonic interfacial wave falls off exponentially with distance on both sides of the interface.

It is assumed that $m'(v_s) \neq 0$ so that $m(\eta)$ may be written as

$$m(\eta) = (\eta - v_s)\tilde{m}(\eta), \tag{53}$$

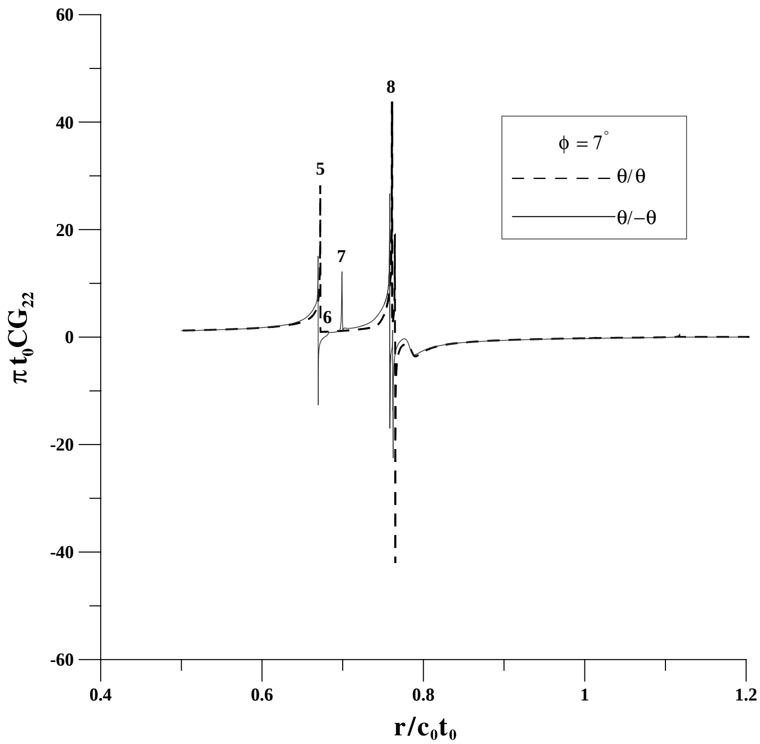

Fig. 5. G_{22} as a function of r for $\phi = 7°$.

where $\tilde{m}(v_s) \neq 0$. It follows from Eq. (53) that

$$\frac{1}{m(\eta)} = \left[\frac{1}{y_1 - v_s} - \mathrm{i}\pi\delta(y_1 - v_s)\right]\frac{1}{\tilde{m}(\eta)}. \tag{54}$$

Substituting Eq. (51) into Eq. (50) and using Eq. (54) yields

$$\dot{\mathbf{u}} = -\frac{1}{\pi(x_1 - v_s t)}\mathrm{Im}[\mathbf{U}(\eta)] + \delta(x_1 - v_s t)\mathrm{Re}[\mathbf{U}(\eta)],$$

where $\mathbf{U}(\eta)$ is given by

$$\mathbf{U}(\eta) = -\frac{\mathrm{adj}(\mathbf{M}(\eta))}{\tilde{m}(\eta)}(\mathbf{M}_2^*(\eta)^{-1}\mathbf{b} + \mathrm{i}\mathbf{F}).$$

Thus upon the arrival of the interfacial wave $\dot{\mathbf{u}}$ exhibits a pole singularity as well as a δ-singularity.

5. Numerical examples

Consider an infinite GaAs crystal, which is of cubic symmetry. The coordinate axes are coincident with the elastic symmetry axes at first. The crystal is cut into two half-crystals so that the interface between the two half-spaces is normal to the x_2-axis. Let the upper half-space be rotated by θ and the lower half-space by $-\theta$ about the normal to the interface; the two half-spaces are then rebonded. The subsonic Stoneley

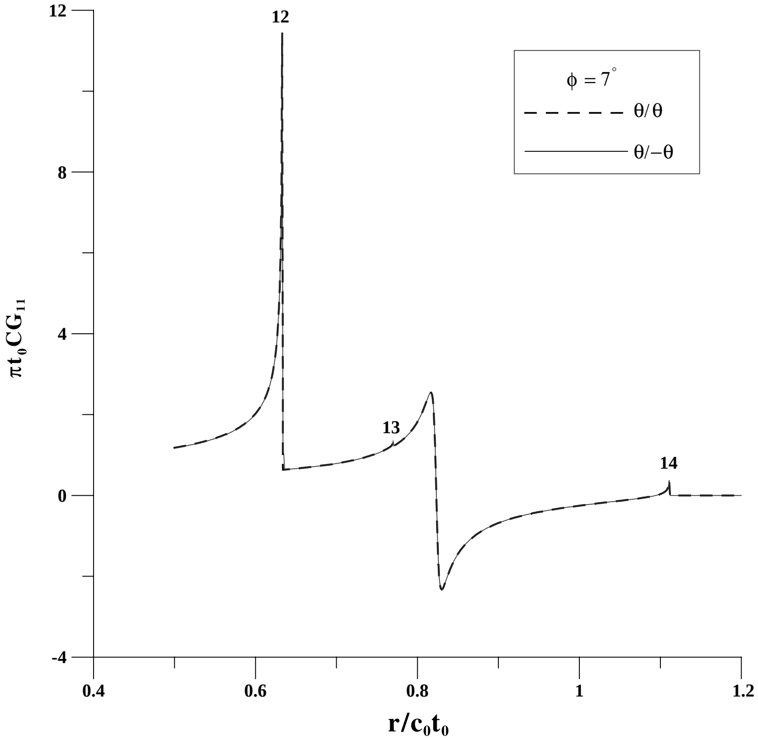

Fig. 6. G_{11} as a function of r for $\phi = 75°$.

wave speeds for such $\theta/-\theta$ bimaterials were computed by Barnett et al. (1985). The elastic constants of GsAs with respect to the symmetry axes in units of 100 GPa are $c_{11} = 1.19$, $c_{12} = 0.538$, $c_{44} = 0.595$, and the mass density is $\rho = 5.31 \times 10^3$ kg/m^3 (Bateman et al., 1959).

The Green's functions G_{11} and G_{22} given by Eq. (48) for $x_2 > 0$ were calculated for a fixed time $t = t_0$ for a 10°/−10° bimaterial. The wave surface of the bimaterial for $x_1 > 0$ is shown in Fig. 1, where the bulk wave fronts are plotted as solid lines and the head wave fronts as dotted lines. The wave surface is symmetric about the interface. The bulk wave fronts are in fact the same as those of a homogeneous 10°/10° material, which is obtained by rotating the whole crystal 10° about the x_2-axis. The additional head waves of the 10°/−10° bimaterial develop because each point at the interface swept by the faster moving bulk waves radiates the more slowly travelling waves. In Fig. 1 only the head wave fronts in the upper half-space are shown. The Green's function G_{11} or G_{22} is expressed in the following dimensionless form:

$$\pi t_0 C G_{\alpha\alpha}\left(\frac{r}{c_0 t_0}\cos\phi, \frac{r}{c_0 t_0}\sin\phi\right), \quad \alpha = 1, 2,$$

where $r = \sqrt{x_1^2 + x_2^2}$, $\phi = \tan^{-1}(x_2/x_1)$, $C = 100$ GPa, and $c_0 = \sqrt{C/\rho} = 4340$ m/s. The Green's functions were calculated as a function of r for $\phi = 0°$, 7°, and 75°. The observation angles and the various wave arrivals indicated by numerals are depicted in Fig. 1. The Green's function G_{11} and G_{22} for a 10°/10° homogeneous material were also computed for comparison purposes.

The results of G_{11} and G_{22} for $\phi = 0$ along the interface are displayed in Figs. 2 and 3, respectively. It can be seen in Fig. 2 that a pseudo-interfacial wave occurs at $r/c_0t_0 = 0.756$ between two bulk waves (points 2 and 3). A plot of the absolute value of $|m|$ as a function of r/c_0t_0 reveals that although $|m|$ does not actually vanish at this pseudo-interface wave speed, its value is a small local minimum. This characteristic feature is

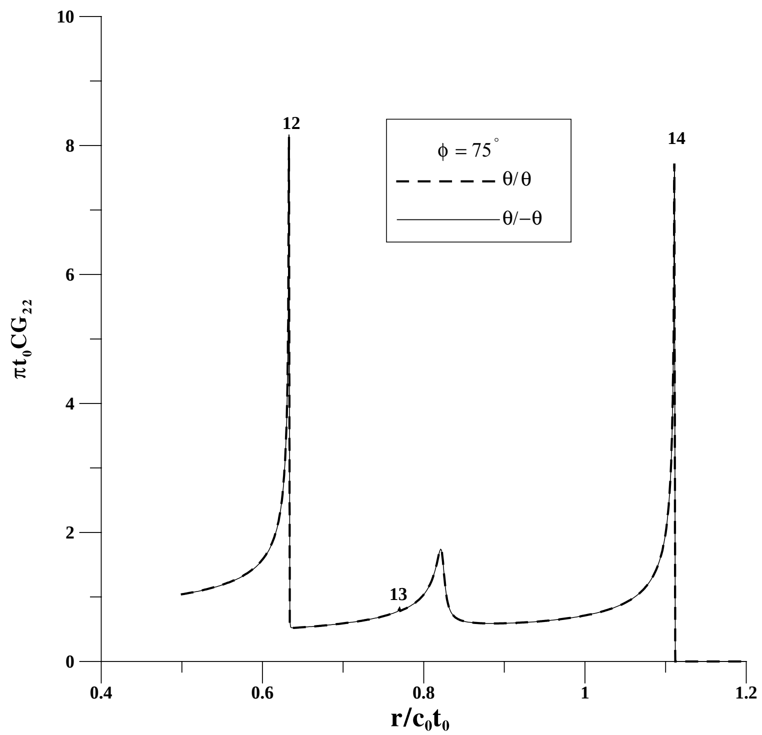

Fig. 7. G_{22} as a function of r for $\phi = 75°$.

similar to that of pseudo-surface waves (Lim and Farnell, 1968). For G_{22} a true subsonic interface wave, denoted by I in Fig. 3, appears at $r/c_0t_0 = 0.709$, which is slightly behind the last bulk wave. The subsonic interface wave speed agrees well with that computed by Barnett et al. (1985). For G_{11} and G_{22} at $\phi = 7°$ and 75° shown in Fig. 4–7, the features of the 10°/–10° bimaterial are essentially the same as those of the 10°/10° homogeneous material except for the presence of the pseudo-interface wave and the head waves. In Fig. 4 for G_{11} at $\phi = 7°$ there are a pseudo-interface wave and two barely visible head waves (points 9 and 10). In Fig. 5 for G_{22} at $\phi = 7°$ there is a pronounced head wave contribution (point 7). In Figs. 6 and 7 the results of the 10°/–10° bimaterial are indistinguishable from those of the 10°/10° homogeneous material.

6. Conclusion

In this paper we have used an extended Stroh's formulation to derive a closed-form solution for a suddenly applied interfacial line force or dislocation in an anisotropic bimaterial. With the extended Stroh's formulation, the solution is obtained without the need of integral transforms. In fact as the extended Stroh's formulation retains the basic structure of the static formulation, the dynamic solution is derived in much the same way as the corresponding static counterpart (Ting, 1996, pp. 278–279).

The solution is used to calculate the response of a bimaterial formed by bonding two misoriented half-spaces of GaAs crystal. It is shown that in addition to true subsonic Stonley waves, pseudo-interface waves are also present. The pseudo-interface waves propagate into either half-space as head waves or shock waves. It is also shown that the response of the bimaterial is essentially the same as that of the homogenous material for large observation angles measured from the interface.

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